1. Define difference of two Sets. Explain with example.

Ans Consider two Sets A and B.
Then the difference between two sets A and B ie $A-B$ can be defined as the elements in Set A. which is not included in Set B.

let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

and $B = \{2, 4, 6, 8, 10, 12, 24, 50\}$.

Then $A - B = \{1, 3, 5, 7, 9\}$.

$B - A = \{12, 24, 50\}$.

$A - B \Rightarrow$ element in A, which are not included in both A and B.

$x \in (A-B) \Rightarrow x \in A$ and $x \notin B$.

$B - A \Rightarrow$ elements in B, which is not included in both A and B.

$x \in (B-A) \Rightarrow x \in B$ and $x \notin A$.

2) if $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$. what bit string represents the set $\{1, 3, 5, 7\}$.

Ans $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

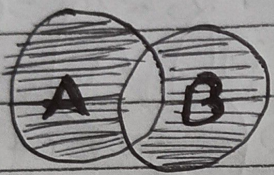
$A = \{1, 3, 5, 7\}$

$= \underline{\underline{10101010}}$

3) Define the Symmetric difference of two sets. Explain with Venn Diagram.

Ans) Symmetric difference between two sets A and B can be defined as.
if $x \in A \oplus B$, then x is either an element of A or either element of B but not an element of both A and B.

if $x \in A \oplus B \Rightarrow (x \in A$ and $x \notin B)$ or
$(x \in B$ and $x \notin A)$



$A \oplus B$

4) ~~What~~ . Define Ceiling function. Explain with an explain

Ans) Ceiling function of x ($\lceil x \rceil$) ~~can be~~ . is the Smallest integer which is greater or equal to x.
it is represented as $\lceil x \rceil$.

eg:- $\lceil 2.8 \rceil = 3$.
$\lceil 5.5 \rceil = 6$
$\lceil 0.1 \rceil = 1$
$\lceil -1.5 \rceil = 1$
$\lceil -0.1 \rceil = 0$

5. What is an invertible function? Give one example.

Ans) if a given function is both one-one and onto. it is said to be invertible function.

eg:- $f(x) = 2x$ where x is an positive integer

$$f(x_1) = f(x_2).$$
$$2x_1 = 2x_2$$
$$\underline{x_1 = x_2}$$ so it is one-one

let $f(x) = 2x = y$.
$$2x = y$$
$$x = y/2$$
$$f(x) = 2 \times \frac{y}{2} = \underline{y}.$$ So it is onto.

6). Give an example of a function which is neither one one nor onto.

Ans). $f(x) = x^2$, where $x \in Z$. $f: Z \to Z$

- $f(x_1) = f(x_2)$.

$x_1^2 = x_2^2$ Since $-1^2 = 1^2 = 1$.

$\pm x_1 = \pm x_2$. ie -1 and 1 have same image 1, So. it is not one one.

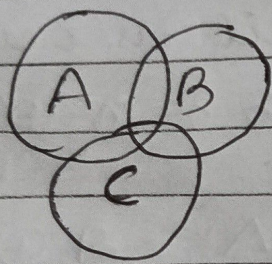
f(x) is not onto since negative numbers and 2, 3, 5, 7, ... does not have a pre-image.

~~7) if A and B are l~~

7) If A, B, C are 3 sets prove that.

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

Ans) ①



$A \cup B \cup C \Rightarrow x \in A$ or B or C

So the ~~no~~ to get the no of element in $A \cup B \cup C$.

we add the no of elements in A and no of elements in B and no of elements in C.

② at that time, the common elements in A and B, A and C, and B and C are added multiple time. to avoid that we substract the no of elements of $(A \cap B)$, no of elements of $A \cap C$ and no of elements of $(B \cap C)$.

(3) By doing so we first ~~[illegible]~~ substract all the common terms. at this time ~~AB~~ The elements common in A and B and C are ~~a~~ not included. So we add ~~that~~ $A \cap B \cap C$ to it.

So finally the equation becomes.

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

8. if f is a function from R to R. with $f(x) > 0$, Show that $f(x)$ is strictly increasing if and only if the function $g(x) = 1/f(x)$ is strictly decreasing.

Ans) Assume that $f(x)$ is increasing

$$\Rightarrow x_1 < x_2 ;$$

$$f(x_1) < f(x_2) . \longrightarrow (1)$$

Assume that $g(x)$ is decreasing.

$$\Rightarrow x_1 < x_2 ;$$

$$g(x_1) > g(x_2) .$$

$$\Rightarrow \frac{1}{f(x_1)} > \frac{1}{f(x_2)} .$$

$$\Rightarrow f(x_1) < f(x_2) . \longrightarrow (2)$$

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from ① and ②
it is clear that $f(x)$ is strictly increasing if and only if the function $g(x) = \frac{1}{f(x)}$ is strictly decreasing.

9). if $A = \{1, 3, 5\}$ and $B = \{1, 2, 3\}$.
find $A \oplus B$.

Ans). $A = \{1, 3, 5\}$
$B = \{1, 2, 3\}$.
$\cancel{A \oplus B = \{2, 5\}}$

$$A \oplus B = (A \cup B) - (A \cap B)$$

or

$$(A - B) \cup (B - A).$$

$(A - B) = \{5\}$
$(B - A) = \{2\}$.
$(A - B) \cup (B - A) = \{5, 2\}$.
$\Rightarrow A \oplus B = \{5, 2\}$.

10). If $S = \{-1, 0, 2, 4, 7\}$.
find $f(S)$, if $f(x) = \lfloor x^2/5 \rfloor$

Ans). $S = \{-1, 0, 2, 4, 7\}$

$f(x) = \lfloor x^2/5 \rfloor$

$f(-1) = \lfloor 1/5 \rfloor = \lfloor 0.2 \rfloor = 0$

$f(0) = \lfloor 0/5 \rfloor = \lfloor 0 \rfloor = 0$

$f(2) = \lfloor 4/5 \rfloor = \lfloor 0.8 \rfloor = 0$

$f(4) = \lfloor 16/5 \rfloor = \lfloor 3.2 \rfloor = \cancel{2}\ 3$

$f(7) = \lfloor 49/5 \rfloor = \lfloor 9.8 \rfloor = 9$

$f(S) = \{0, 3, 9\}$

11). Prove that $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$

Ans). ~~a). let $x \in (A \cup B)$~~

a) $(A \cup B)^c = A^c \cap B^c$

let $x \in (A \cup B)^c \Rightarrow x \notin A \cup B$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A^c$ and $x \in B^c$

$\Rightarrow x \in A^c \cap B^c$

Since x is an arbitrary element every element of $(A \cup B)^c$ is an element of $(A^c \cap B^c)$

$\Rightarrow (A \cup B)^c \subset (A^c \cap B^c) \longrightarrow ①$

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let $y \in A^c \cap B^c \Rightarrow y \in A^c$ and $y \in B^c$

$\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin (A \cup B)$

$\Rightarrow y \in (A \cup B)^c$

Since y is an arbitrary element, every element of $A^c \cap B^c$ is an element of $(A \cup B)^c$

$\Rightarrow A^c \cap B^c \subset (A \cup B)^c \longrightarrow ②$

from ① and ②.

$$(A \cup B)^c = A^c \cap B^c$$

b). $(A \cap B)^c = A^c \cup B^c$

let $x \in (A \cap B)^c \Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ or $x \notin B$.

$\Rightarrow x \in A^c$ or $x \in B^c$

$\Rightarrow x \in (A^c \cup B^c)$.

Since x is an arbitrary element, every element of $(A \cap B)^c$ is an element of $A^c \cup B^c$.

$\Rightarrow (A \cap B)^c \subset (A^c \cup B^c) \longrightarrow ①$

let $y \in (A^c \cup B^c) \Rightarrow y \in A^c$ or $y \in B^c$

$\Rightarrow y \notin A$ ~~and~~ or $y \notin B$

$\Rightarrow y \notin A$ and B.

$\Rightarrow y \in (A \cap B)^c$

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Since y is an arbitrary element every element of $A^c \cup B^c$ is an element of $(A \cap B)^c$

$\Rightarrow A^c \cup B^c \subset (A \cap B)^c \Rightarrow$ (2)

from (1) & (2)

$\Rightarrow$ ~~$A^c \cup B^c$~~ $(A \cap B)^c = A^c \cup B^c$

12. Check whether the following functions are invertible or not. If it is invertible find the inverse.

a) f from $R \to R$. $f(x) = x^2$

b) $f: R \to R$ $f(x) = 7x + 12$

Ans) a) $f: R \to R$ $f(x) = x^2$

$f(x_1) = f(x_2)$

$x_1^2 = x_2^2$

$\pm x_1 = \pm x_2$

ie +1 and -1 have same image 1, same in case of +2 & -2. and so on. So $f(x) = x^2$ is not one one function. Since it is not one one, the function is not invertible.

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b). $f: R \to R$ $f(x) = 7x + 12$

$$f(x_1) = f(x_2)$$
$$7x_1 + 12 = 7x_2 + 12$$
$$7x_1 = 7x_2$$
$$x_1 = x_2$$

Since $x_1 = x_2$, it is one one.

let $f(x) = y$.

$$\Rightarrow 7x + 12 = y$$
$$7x = y - 12$$
$$x = \frac{y-12}{7}$$

$$f(x) = 7x + 12$$
$$= \frac{7(y-12)}{7} + 12$$
$$= y - 12 + 12$$
$$= y$$

Since $f(x) = y$. $f(x)$ is onto

Since function is both one one and onto. it is invertible function.

So $f^{-1}(x) = \frac{x-12}{7}$